**BELLCOMM, INC.**

SUBJECT: Magnetic Moments and Torques on the AAP Cluster Configuration - Case 620

DATE: December 4, 1967

FROM: D. P. Woodard

## ABSTRACT

The torque created by the interaction of a spacecraft magnetic moment and the earth's magnetic field must be countered by the control moment gyro stabilization system on AAP-3 and AAP-4. Since the magnetic torque is small in absolute value, compared to gravity-gradient torque, the associated periodic component of momentum is of no importance. Interest in the magnetic case is limited, therefore, to bias momentum effects which may lead to significant propellant requirements for desaturating the gyros. Lack of definition of the ferromagnetic material in the configuration makes it impossible to form precise estimates of these effects.

What is felt to be a conservative, worst-case analysis has been effected for this configuration by making gross estimates of ferromagnetic quantities and their associated spacial distributions. Using an assumed 1% of total system weight for the ferromagnetic material (1300 pounds), the vehicle magnetic moment is estimated at 7000 ampere-meter$^2$. The maximum, average-per-orbit torque tending to align this moment with the earth's magnetic field is .12 lb-ft for a 240 nm, 28.5° inclined orbit. The resulting bias momentum is comparable to that due to the gravity-gradient torque acting on the AAP-3 and AAP-4 cluster with its long axis in the orbital plane.

A discussion is included on the nature of the earth's magnetic field and basic magnetic concepts as a basis for these estimates.

(NASA-CR-93090) MAGNETIC MOMENTS AND TORQUES ON THE AAP CLUSTER CONFIGURATION (Bellcomm, Inc.) 16 p

N79-72076

Unclas

00/15 11048

**BELLCOMM, INC.**
1100 Seventeenth Street, N.W. Washington, D.C. 20036

**SUBJECT:** Magnetic Moments and Torques on the AAP Cluster Configuration - Case 620

**DATE:** December 4, 1967

**FROM:** D. P. Woodard

MEMORANDUM FOR FILE

## INTRODUCTION

Recent literature has been attentive to the interaction of an orbiting satellite with the earth's magnetic field. In particular the authors have considered:

(1) The reduction of the angular velocity of a spinning satellite as a result of eddy currents induced by its motion relative to the earth's field.

(2) The disturbing torques arising from the natural inclination of a magnetized vehicle to align its magnetic axis with the earth's field.

(3) The possible utilization of these interactions for vehicle stabilization and control.

These studies have considered vehicles which are reasonably amenable to definition of their magnetic properties. In general, since the vehicles have been relatively small, it has been possible to consider individual magnetic elements and components, to make direct measurements before flight on the assembled operating structure, and, on occasion for spin-stabilized vehicles, to deduce effective moments from flight data. The several AAP missions together will result in the assembly of a large vehicle in earth orbit which presents a much more complex structural and magnetic problem. The assembled configuration is unique in size and in its mission requirements for attitude and pointing stability. It will be susceptible to magnetic torque disturbances since it is a conducting body moving through a magnetic field, and since its magnetic elements will combine to produce a net magnetic moment.

This memorandum discusses the earth's field, the computation of magnetic moments and torques, and attempts to establish an upper bound for the several torque components.

THE EARTH'S MAGNETIC FIELD

The earth's field in space occupied by an orbiting satellite can be approximated with a combination of one or more magnetic dipoles located at or near the center of the earth [1]. This memorandum considers only the simplest, first approximation, that of a single dipole aligned with the earth's magnetic axis with its north-seeking-end directed toward the southern magnetic pole. To develop a quantitative feel, we recall that the flux density at any point, P, in the vicinity of a magnetic dipole is given by [11]:

$$\bar{B} = -\frac{\mu_0}{4\pi}\left[\frac{\bar{m}}{r^3} - 3\,\frac{(\bar{m}\cdot\bar{r})}{r^5}\,\bar{r}\right]\ \frac{\text{webers}}{\text{meter}^2} \tag{1}$$

Using the rationalized MKS system of units, the moment, $\bar{m}$, describes the dipole strength in amperes-meters$^2$. $\bar{r}$ is the radius vector in meters joining P with the dipole center, as shown in Figure 1A, and intersecting $\bar{m}$ at an angle $\psi$. $\mu_0$ is the permittivity of free space, $4\pi \times 10^{-7}$ webers/ampere-meter. In spherical components, (1) resolves into

$$B_r = -\frac{\mu_0 m}{2\pi r^3}\cos\psi$$

$$B_\psi = \frac{\mu_0 m}{4\pi r^3}\sin\psi$$

$$B_\theta = 0 \tag{2}$$

where r, $\psi$, and $\theta$ are as shown in Figure 1B. For the earth, $|\bar{m}|$ is approximately $.81 \times 10^{23}$ ampere-meter$^2$; taking $r = 6.371 \times 10^6$ meters, $\frac{\mu_0 m}{2\pi r^3}$ has a magnitude of about $.62 \times 10^{-4}$ webers/meter$^2$. Maximum and minimum flux densities occur where $\sin\psi \cos\psi = 0$, that is, over the magnetic poles and equator, respectively. One may visualize two simple cases: The field is constant and unidirectional in the magnetic equatorial plane where $\psi = \frac{\pi}{2}$; in the magnetic polar plane the field is periodic, being composed of the two periodic components $B_r$ and $B_\psi$.

Since the earth's magnetic and spin axes are inclined at an angle of about 12 degrees, a vehicle in any practical orbital plane will be subject to a field which varies both in magnitude and direction as a function of its position. To determine the effects of the field on a vehicle, successive coordinate transformations are necessary, therefore, to translate from the earth fixed spherical system of Equation (2) to a rectangular inertial system. Hodge and Blackshear [2] have performed this transformation and have integrated the components of $\overline{B}$ over complete orbital periods for various orbital altitudes and inclinations. Their results permit computation of the average, or bias, torque exerted on an inertially oriented vehicle by the earth's field on a per orbit basis. Figure 2 shows the average flux density vector, $\overline{B}_A$, in an ecliptic system. $\overline{B}_A$ is inclined at an angle, $\Gamma_A$, with respect to the ecliptic normal about which it rotates annually. The dependence of $\overline{B}_A$ on orbit inclination, i, and altitude, r, can be expressed from their results by

$$|\overline{B}_A| = \frac{\mu_0 m}{16\pi r^3}[3 + \cos 2i] \; \frac{\text{webers}}{\text{meter}^2} \tag{3}$$

The angular position of $\overline{B}_A$ with respect to the ecliptic normal is the sum of two components, an average angle depending on orbit inclination

$$\Gamma_{AM} = 22.5 + 66.4 \; [1 - \cos (2i - 15)] \text{ degrees} \tag{4}$$

and a periodic, or ripple, component given by

$$\Gamma_{AP} = 28.5 \cos \frac{2\pi}{T} t \text{ degrees} \tag{5}$$

The period, T, is a function of the orbit nodal regression rate, $\Omega$,

$$T = \frac{2\pi}{\Omega} = 36 \left[ \left(\frac{a}{r}\right)^{7/2} \cos i \right]^{-1} \text{days}$$

and a is the earth radius. Typically for a 240 nm orbit inclined at 28.5 degrees, we find:

$$|\bar{B}_A| = .227 \times 10^{-4} \text{ webers/ meter}^2$$

$$\Gamma_{AM} = 37.5 \text{ degrees}$$

$$\Gamma_{AP} = 28.6 \text{ degrees}$$

$$T = 52 \text{ days}$$

An instantaneous time description of $\bar{B}$ is not given by Hodge and Blackshear in ecliptic coordinates. However to estimate the maximum effect of torque disturbances, we note from (2) that the earth's flux density will always be less than the peak value which occurs directly over a magnetic pole.

## MAGNETIC MOMENTS AND TORQUES

A body having a magnetic moment and in a magnetic field is subject to a torque given by

$$\bar{T} = \bar{m} \times \bar{B} \text{ newton meters}$$

where $\bar{m}$ is the equivalent dipole moment of the body and $\bar{B}$ is the field flux density. The moment may be acquired in a number of ways.

Current Flow

The concept of magnetic moment is usually defined in terms of a plane coil carrying a current, I, and enclosing an area, A, i.e.,

$$\bar{m} = \bar{n} \text{ IA amperes-meter}^2 \tag{8}$$

where $\bar{n}$ is the unit vector normal to A (right hand rule). This approach is useful for evaluating moments associated with current carrying conductors, as in an electrical power system, and as a basis for computing torques associated with circulating currents induced in a conducting body moving with respect to a magnetic field. The latter case is formulated by Smith [20] who shows that the torque on a thin wall cylinder spinning in a uniform magnetic field normal to its axis is given by

$$|T| = \pi \ \sigma \ B^2 \omega \ a^3 \ell \ h\left(1 - \frac{2a}{\ell} \tanh \frac{\ell}{2a}\right) \text{ Newton-meters} \tag{9}$$

where

a = radius in meters

$\ell$ = length in meters

h = thickness in meters

$\sigma$ = conductivity in (ohm-meters)$^{-1}$

His derivations also show that the torque on a cylinder tumbling end-over-end in a similar field, when averaged over a revolution, is exactly half that given by (9).

Permanent and Induced Magnetic Moment

The state of magnetization of a body (or magnetic moment per unit volume) is defined by

$$\bar{M} = \frac{\bar{B}_i}{\mu_o} - \bar{H}_i \text{ amperes/meter} \tag{10}$$

where $\bar{B}_i$ is the internal flux density and $\bar{H}_i$ is the internal magnetizing force. The total moment is obtained by integrating over the complete volume, V,

$$\bar{m} = \int_V \bar{M}\, dV = \int_V \left(\frac{\bar{B}_i}{\mu_0} - \bar{H}_i\right) dV \text{ amperes-meter}^2 \tag{11}$$

The magnetic moment per unit volume can only be constant for an ellipsoid so that for other shapes (11) is usually written

$$\bar{m} = \left(\frac{\bar{B}_i}{\mu_0} - \bar{H}_i\right) A\ell\ R_S \tag{12}$$

where A is the body cross-sectional area, $\ell$ is the length, and the shortening factor, $R_S$, includes the geometrical effects. Typically for cylinders having length to diameter ratios, $(\frac{\ell}{d})$, greater than 10, $R_S$ approaches .75 asymptotically [19].

An ideal magnetic material is one in which $\bar{M}$ and $\bar{B}_i$ in (10) are in the same direction and related by a constant of proportionality called the susceptibility, $\chi$, or,

$$\bar{M} = \chi \frac{\bar{B}_i}{\mu_0} \tag{13}$$

Combining (13) and (10) gives the familiar expression and definition of permeability, $\mu$,

$$\bar{B}_i = \frac{\mu_0}{1-\chi} \bar{H}_i = \mu_0 \mu_r\ \bar{H}_i = \mu \bar{H}_i \tag{14}$$

Since $\mu_r$ is large for good magnetic materials, $\frac{\bar{B}_i}{\mu_0} >> \bar{H}_i$, and the moment of a cylindrical body is closely

$$\bar{m} = \frac{\bar{B}_i}{\mu_0} A\ell \; R_S \text{ amperes/meter}^2 \qquad (15)$$

Finally the relation between internal and external magnetizing forces and moment per unit volume is given by

$$\bar{H}_i = \bar{H}_0 - D_B\left(\frac{\bar{B}_i}{\mu_0} - \bar{H}_i\right)\frac{\text{amperes}}{\text{meter}} \qquad (16)$$

Values for the demagnetizing factor, $D_B$, have been computed for various shapes and range from $1.4 \times 10^{-2}$ to $3.3 \times 10^{-4}$ for cylinders where $10 \leq \frac{\ell}{d} \leq 100$ [19]. In a permanent magnet, the magnetizing force $\bar{H}_i$, is generated within the magnet itself; $\bar{H}_0 << \bar{H}_i$, and the state of magnetization to compute moment in (12) can be determined from the material (B-H) characteristic and $D_B$. For induced moment, $\bar{H}_0$ arises externally; $\bar{H}_i << \bar{H}_0$, and $(\frac{\bar{B}_i}{\mu_0} - \bar{H}_i)$ and $\bar{H}_0$ are related by $D_B$ up to where the material enters saturation. To compute moment in either case some knowledge of materials and geometry is necessary.

## ESTIMATION OF MOMENT AND TORQUE MAGNITUDES

A literature search has produced very little data from which realistic magnitudes of moments and torques can be extrapolated for the AAP-type vehicle configurations. Table I summarizes all mention of moments so far encountered, whether arising from direct measurement, data reduction, or hypotheses. On this basis we can only expect to make coarse upper-bound type estimates, utilizing the preceding field, moment and torque concepts. Consider therefore a cylindrical vehicle in orbit with a gross weight of 130,000 pounds of which 1 percent by weight is composed of ferromagnetic materials. Assume further that the assembly has a length of 100 feet and a diameter of 22 feet. Taking the density of iron as 7.9 grams/cm$^3$, we compute a volume of magnetic material of about .075 meter$^3$. Little use of magnetic material is expected in the spacecraft structure. Most magnetic material present will likely be in the form of electrical components (motors, transformers, relay armatures, etc.) which should neither produce large moments individually nor create an effective magnet when considered together. In most cases magnetic material used for these purposes is characterized by a relatively narrow hysteresis loop having a coercive force in the order of 100 amperes/meter, comparable to annealed pure iron. This should be a realistic estimate for $|\bar{H}_i|$ in (16). An effective $\frac{\ell}{d}$ ratio of 50 with a corresponding demagnetizing factor of 1/800 appears to be a reasonable compromise for an effective geometry. Using these values in (16), we compute an effective moment of 4500 ampere-meter$^2$ and an average torque acting throughout each 240 nm, 28.5 degree orbit of .077 lb-ft.

In weak fields the relative permeability of demagnetized iron is about 200. This assures that $\bar{H}i << \bar{H}o$ in Equation (16) and using the previous geometry, we compute an equivalent induced moment of 815 ampere-meter$^2$ with a corresponding average bias torque of .014 lb-ft.

The power supply current is a potential moment contributor as shown by Equation (8). For example, a 28 volt power system delivering 5KW to a load 100 feet away over conductors separated by 1 foot will produce a moment of 1700 ampere-meter$^2$. The corresponding average per orbit torque is .03 lb-ft and the maximum peak torque is .06 lb-ft. Considering the fact that the effective moment is the vector sum of the individual spacecraft circuits, most of which have conductor separations much less than 1 foot, this estimate should be conservative by a substantial factor.

The torque contribution from induced currents as the assumed assembly moves through the earth's field, Equation (9), is small due to the low rate of flux change through the cylinder. Since the vehicle is not spinning about an axis, its equivalent maximum angular rate in the earth's field is $2\pi$ radians per orbit which occurs in the magnetic polar plane. Assuming a 90 minute orbital period, the same cylindrical dimensions previously assumed, a shell thickness of 0.1 inch, and resistivity of $2.8 \times 10^{-8}$ ohm- meters for aluminum, the equivalent average per orbit torque is $1.51 \times 10^{-4}$ lb-ft and the peak torque is $7.66 \times 10^{-4}$ lb-ft.

SUMMARY AND CONCLUSIONS

For convenience we tabulate our estimated upper bounds as follows

| Source | Moment amp-meter$^2$ | Upper Bound Average torque lb ft |
|---|---|---|
| Earth Induction | 815 | .014 |
| Residual Magnetism | 4500 | .077 |
| Power Currents | $1.7 \times 10^3$ | .030 |
| Induced Currents | - | $1.51 \times 10^{-4}$ |
| Arithmetic Sum | | .12 |

This tabulation shows that the torque components due to induction by the earth's field, unbalanced power supply currents, and circulating currents induced by motion through the earth's field are smaller and of less potential importance than the effect that might be produced if a relatively large mass of permanently magnetized material is incorporated in the orbiting assembly. The assumptions under which these smaller torque components are estimated have been chosen to produce conservative upper bounds. The residual magnetism component is much less definite in this respect. In view of its magnitude, further study of the mass, volume, geometry and state of magnetization of the magnetic components included in the vehicle is advisable so that an adequate magnetic model can be developed.

D. P. Woodard

D. P. Woodard

1022-DPW-ajl

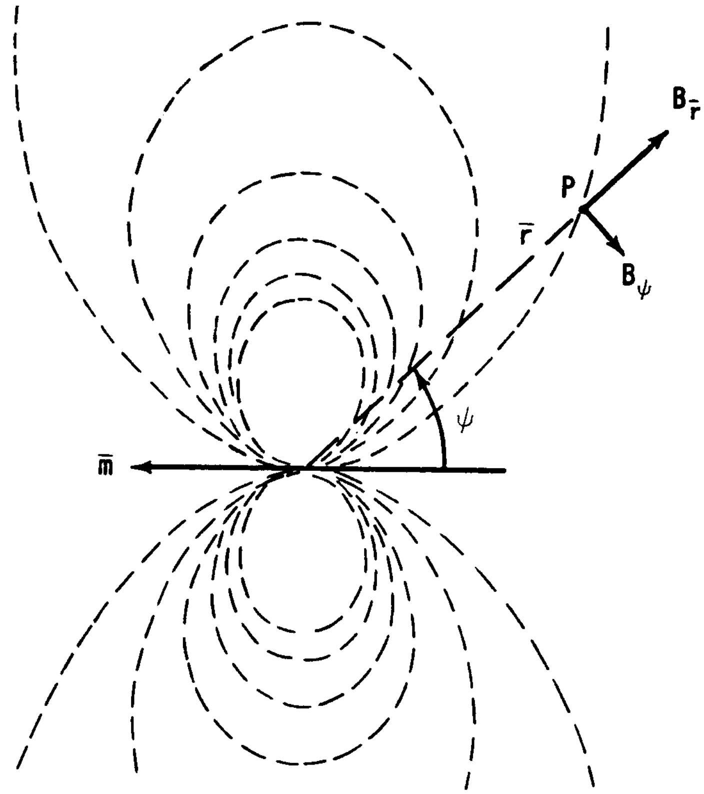
Attachments
Table 1
Figures 1 and 2
Bibliography

TABLE I

Literature Reported Moments

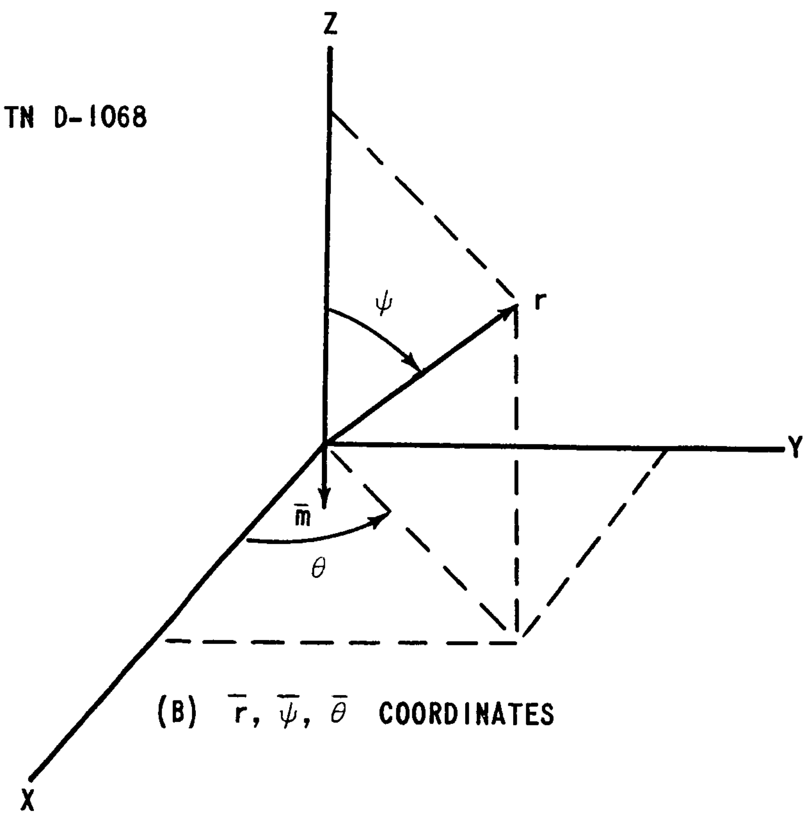
| Reference* | Vehicle | Moment amp-meter$^2$ |
|---|---|---|
| [7] | Snap 10A reactor | 95 |
| | Thermo-converter | 15 |
| [6] | OAO, coil designed for control purposes | 14.3 |
| [12] | 10kg = 22 lb of ferromagnetic material | 8.6 |
| [13] | Tiros I(due mainly to spin) | .896 |
| [14] | designed permanent magnet for 1 lb ft torque for large manned space station | 38,700 |
| [4] | coil designed for control of a 3 ft spherical vehicle | 93 |
| [15] | 3000 lb vehicle using a 30 lb Alnico V magnet for control | 1160 |
| [16] | Explorer XI | 2.35 |
| | Explorer IV | 19 |
| [17] | 21,000 lb vehicle | 1160 |

*See Bibliography



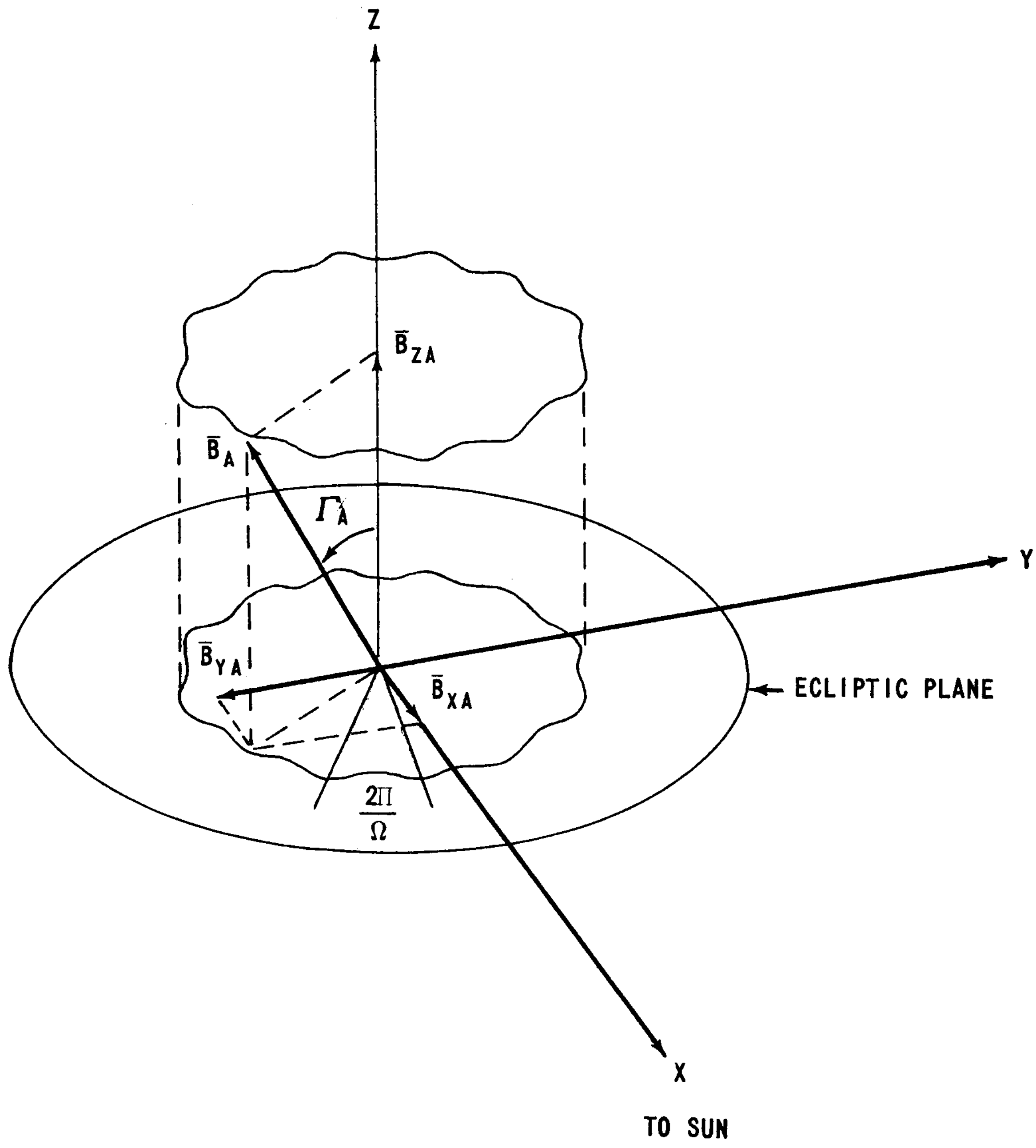

(A) A MAGNETIC DIPOLE FIELD*

*SKETCHED FROM NASA TN D-1068




(B) $\bar{r}$, $\bar{\psi}$, $\bar{\theta}$ COORDINATES

FIGURE 1



Z
$\bar{B}_{ZA}$
$\bar{B}_A$
$\Gamma_A$
Y
$\bar{B}_{YA}$
$\bar{B}_{XA}$
ECLIPTIC PLANE
$\frac{2\Pi}{\Omega}$
X
TO SUN


FIGURE 2

# BIBLIOGRAPHY


1. McNish, A. G., "On Causes of the Earth's Magnetism and its Changes," Terrestial Magnetism and Electricity, McGraw-Hill, New York, 1939, p. 308.
2. Hodge and Blackshear, "An Analytical Study of the Magnetic Field Encountered by Artificial Earth Satellites in Circular Orbits," NASA TN D-2041. February 1964.
3. Minkus, W. J., "A Method for Controlling the Attitude of a Satellite Using the Earth's Magnetic Field," RADC-TDR-64-394. September 1964.
4. Wheeler, P. C., "Magnetic Attitude Control of Rigid, Axially Symmetric, Spinning Satellites in Circular Earth Oribt," NASA CR-313. October 1965.
5. Smith, G. L., "Effects of Magnetically Induced Eddy-Current Torques on Spin Motions of an Earth Satellite," NASA TN D-2198, April 1965.
6. White, Shigemato, Bourguin, "Satellite Attitude Control Utilizing the Earth's Magnetic Field," NASA TN D-1068, August 1961.
7. Boyle and Galantine, "Magnetic Moments and Vehicle Torques in the Snap 10A," NAA-SR-11207.
8. Fonov, Yu. V., "On the Problem of the Interaction Between a Satellite and the Earth's Magnetic Field", NASA TT F-37, May 1960.
9. Shortley and Williams, Physics, Vol. II, Prentice-Hall, New York, 1950.

10. Smythe, Static and Dynamic Electricity, McGraw-Hill, New York, 1950.

11. Becker-Sauter, Electromagnetic Fields and Interactions, Volume I, Blaisdell, New York, 1964.

12. Spitzer, Lymann, "Space Telescopes and Components", The Astronomical Journal, Vol. 65, No. 5, June 1960.

13. Bandeen and Manger, "Angular Motion of the Spin Axis of the Tiros I Meteorological Satellite Due to Magnetic and Gravitational Torques", NASA TN D-571, April 1961.

14. Langley Research Center Staff, "A Report on the Research and Technological Problems of Manned Rotating Spacecraft", NASA TN D-1504, August 1962.

15. Adams and Brissenden, "Satellite Attitude Control using a Combination of Inertia Wheels and a Bar Magnet", NASA TN D-626, November 1960.

16. Naumann, R. J., "Observed Torque-Producing Forces Acting on Satellites", NASA TR R-183, October 1963.

17. Keeler, Jack L., "Satellite Disturbance Torques," Source unknown.

18. Bozorth, R. M., Ferromagnetism, D. Van Nostrand Co, Inc., New York, 1951.

19. Glass, M. S., "Principles of Design of Magnetic Devices for Attitude Control of Satellites", Bell System Technical Journal, Volume XLVI, No. 5, May - June 1967.

20. Smith, G. Louis, "A Theoretical Study of the Torques Induced by a Magnetic Field on Rotating Cylinders and Spinning Thin-Wall Cones, Cone Frustrums, and General Body of Revolution," NASA TR R-129, 1962.

BELLCOMM, INC.

Subject: Magnetic Moments and Torques on the AAP Cluster Configuration - Case 620

From: D. P. Woodard

## Distribution List

NASA Headquarters

Messrs. H. Cohen/MLR
P. E. Culbertson/MLA
J. H. Disher/MLD
J. A. Edwards/MLO
L. K. Fero/MLV
J. P. Field, Jr./MLP
T. A. Keegan/MA-2
C. W. Mathews/ML
M. Savage/MLT

Bellcomm

Messrs. A. P. Boysen
D. R. Hagner
W. C. Hittinger
B. T. Howard
J. Z. Menard
I. D. Nehama
I. M. Ross
R. L. Wagner
Div. 101 Supervision
All Members Dept. 1021, 1022, 1024
Department 1023
Central File
Library

MSFC

Messrs. L. F. Belew/I-S/AA
N. R. Gilino/R-ASTR-S
G. B. Hardy/I-S/AA
F. E. Vruels/I-I/IB-E

MSC

Messrs. H. W. Dotts/KS
W. B. Evans/KM
W. J. Klinar/EG
K. L. Lindsay/EG23
R. F. Thompson/KA